EIGENFILTERS FOR SIGNAL CANCELLATION

Sunil Bharitkar and Chris Kyriakakis

Immersive Audio Laboratory
University of Southern California
Los Angeles. CA 90064. USA
Phone:+1-213-740-8600 Fax:+1-213-740-4651,
Email:ckyriak@imsc.edu.edu,bharitka@sipi.usc.edu

Abstract: Selectively canceling signals at specific locations within an acoustical environment with multiple listeners is of significant importance for home theater, automobile, teleconferencing, office, industrial and other applications. The traditional noise cancellation approach is impractical for such applications because it requires sensors that must be placed on the listeners. In this paper we investigate the theoretical properties of eigenfilters for signal cancellation proposed in [1]. We also investigate the sensitivity of the eigenfilter as a function of the room impulse response duration. Our results show that with the minimum phase model for the room impulse response, we obtain a better behaviour in the sensitivity of the filter to the duration of the room response.

1. INTRODUCTION

Selective signal cancellation is required in applications that require a signal of interest to be enhanced while minimizing the effects of noise or other signals. For example, in home theater or television viewing applications a listener in a specific position in a room may not want to listen to the audio signal being transmitted, while another listener at a different position would prefer to listen to the signal. Consequently, if the objective is to keep one listener in a region with a reduced sound pressure level, then one can view this problem as that of signal cancellation in the direction of that listener. Similar applications arise in the automobile or any other environment with multiple listeners in which only a subset wish to listen to the audio signal.

In this paper we investigate the theoretical aspects of the filter proposed in [1] - where we viewed the signal cancellation problem as a maximization of the difference in the received power at two different locations. Specifically we have not considered the perceptual (psychoacoustical) viewpoint (where an added condition would be to satisfy some listening requirement in a given direction). Any signal cancellation methodology must inevitably take into account aspects of human perception in the given environment. We shall consider this in forthcoming papers. In the next section, we briefly discuss the proposed eigenfilter method. In Section 3, we discuss the algebraic and linear phase properties of the proposed filter. We investigate the performance of the filter as a function of the room impulse response duration. We also show in section 4, that, in general an eigenfilter based on the minimum phase model of the room responses provides a better sensitivity behaviour. Section 5 concludes the paper.

2. INFORMATION THEORY MODELS

In [1], we proposed a filter based on optimizing the mutual information function. Mutual information $I(x,y)$ (MI) measures arbitrary dependencies between two random variables x, and y with marginal distributions denoted by $f(x)$, and $f(y)$, and their joint distribution $f(x,y)$ (where x may be considered as an input to a channel, and y is the corresponding output). The general form for this measure is,

$$I(x,y) = \sum_x \sum_y f(x,y) \log \frac{f(x,y)}{f(x)f(y)} \tag{1}$$

$I(x,y) \geq 0$, with equality being achieved on general statistical independence between x, and y, and $I(x,y) = I(y,x)$. We showed that in the presence of a linear time invariant channel $h(n)$-representing the room impulse response, and additive white gaussian noise (AWGN) $v(n)$, the mutual information can be rewritten as

$$I(y(n), x(n)) = \frac{1}{2} \log(\frac{\sigma_y^2(n)}{\sigma_v^2(n)}) \tag{2}$$

where, $\sigma_y^2(n) = r_y(0) = E\{|y(n)|^2\}$, and $\sigma_v^2(n)$ represents the average power of the additive white gaussian noise.

As mentioned above, we view the signal cancellation problem as a maximization of the difference in the received power in dB at two different locations, with their corresponding room impulse responses denoted as $h_1(n)$, and

$h_2(n)$. Thereby, we proceeded to show that the objective function for this problem can be cast as (with the fact that the $\log(x)$ is a monotonic function in (2)),

$$J(n) = max_{\underline{w}} \quad \frac{1}{2}(\frac{\sigma^2_{y_2(n)}}{\sigma^2_{v_2(n)}}) - \lambda(\frac{\sigma^2_{y_1(n)}}{\sigma^2_{v_1(n)}} - \psi) \qquad (3)$$

which pertains to maximization of the signal power in the direction of listener 2, while retaining the power towards listener 1 at least $10^{\psi_{dB}/10}$, and $\sigma^2_{v_i(n)} = E\{|y_i(n)|^2\}, i = \{1,2\}$; with

$$\begin{aligned} y_i(n) &= h_i(n) \star (\sum_{k=0}^{M-1} w_k x(n-k)) + v_i(n); \quad i = 1,2 \\ \underline{w} &= (w_0, w_1, ..., w_{M-1})^T \end{aligned} \qquad (4)$$

where, $\star$ denotes convolution, and $\underline{w}$ are the filter coefficients. The solution to the constrained optimization problem (3) is the optimal FIR filter, $\underline{w}^* \in \Re^{M \times 1}$ given by,

$$\underline{w}^* = \underline{e}_{\lambda_{max}[B^{-1}A]} \qquad (5)$$

where, $\underline{e}_{\lambda_{max}[\phi]}$ is the eigenvector corresponding to the maximum eigenvalue of the matrix ϕ, with,

$$\begin{aligned} A &= \sum_{p=0}^{L-1}\sum_{q=0}^{L-1} h_2(p)h_2(q)\mathbf{R}_{\underline{x}}(p,q) \\ B &= \sum_{p=0}^{L-1}\sum_{q=0}^{L-1} h_1(p)h_1(q)\mathbf{R}_{\underline{x}}(p,q) \end{aligned} \qquad (6)$$

The correlation matrix $\mathbf{R}_{\underline{x}}(p,q) \in \Re^{M \times M}$ is defined by,

$$\begin{aligned} \mathbf{R}_{\underline{x}}(p,q) &= E\{\underline{x}(n-p)\underline{x}^T(n-q)\} \\ \underline{x}(n-l) &= (x(n-l), .., x(n-l-M+1))^T \end{aligned} \qquad (7)$$

The gain in dB can be computed by,

$$\begin{aligned} G_{dB} &= 10\log_{10}\frac{\sigma^2_{y_2(n)}}{\sigma^2_{y_1(n)}} \\ &= 10\log_{10}\frac{\underline{w}^{*T}A\underline{w}^*}{\underline{w}^{*T}B\underline{w}^*} \end{aligned} \qquad (8)$$

For a WSS process the matrix $\mathbf{R}_{\underline{x}}(p,q)$ is toeplitz, and the gain can be expressed as,

$$G_{dB} = 10\log_{10}\frac{\int_{2\pi}|W^*(e^{j\omega})|^2|H_2(e^{j\omega})|^2 S_x(e^{j\omega})d\omega}{\int_{2\pi}|W^*(e^{j\omega})|^2|H_1(e^{j\omega})|^2 S_x(e^{j\omega})d\omega} \qquad (9)$$

where, $r_x(k) \in \mathbf{R}_{\underline{x}}(k)$ and $S_x(e^{j\omega})$ form a fourier transform pair.

In the next section we shall characterize this eigenfilter in terms of its mathematical properties.

3. PROPERTIES OF EIGENFILTERS

In this section, we state and prove some theoretical properties of the eigenfilter (5). Below, we provide some definitions and properties pertaining to doubly symmetric matrices, which toeplitz matrices are a special case of.

Definition 1: A $p \times p$ doubly symmetric matrix $\mathbf{Q}$ satisfies the following relation,

$$\mathbf{Q} = \mathbf{JQJ} \qquad (10)$$

where, $\mathbf{J}$ is a diagonal matrix with unit elements along the northeast-southwest diagonal. Basically, premultiplying (postmultiplying) a matrix with $\mathbf{J}$ exchanges the rows (columns) of the matrix.

Property 1: A scaling term - c, associated with a doubly symmetric matrix leaves its doubly symmetricity unaltered. This can be easily seen as follows,

$$\mathbf{J}c\mathbf{QJ} = c\mathbf{JQJ} = c\mathbf{Q} \qquad (11)$$

Property 2: Linear combination of doubly symmetric matrices yields a doubly symmetric matrix.

$$\begin{aligned} \mathbf{J}[c_1\mathbf{Q}_1 + c_2\mathbf{Q}_2]\mathbf{J} &= c_1\mathbf{JQ}_1\mathbf{J} + c_2\mathbf{JQ}_2\mathbf{J} \\ &= c_1\mathbf{Q}_1 + c_2\mathbf{Q}_2 \end{aligned} \qquad (12)$$

Hence, from the above properties, the matrices A, and B are doubly symmetric.

Property 3: The inverse of a doubly symmetric matrix is doubly symmetric.

$$\begin{aligned} \mathbf{Q} &= \mathbf{JQJ} \\ \mathbf{Q}^{-1} &= (\mathbf{JQJ})^{-1} = \mathbf{J}^{-1}\mathbf{Q}^{-1}\mathbf{J}^{-1} = \mathbf{JQ}^{-1}\mathbf{J} \end{aligned} \qquad (13)$$

Property 4: The product of doubly symmetric matrices is doubly symmetric.

$$\mathbf{Q}_1\mathbf{Q}_2 = \mathbf{JQ}_1\mathbf{JJQ}_2\mathbf{J} = \mathbf{JQ}_1\mathbf{Q}_2\mathbf{J} \qquad (14)$$

where, we have used the fact that $\mathbf{JJ} = \mathbf{J}^2 = I$. Thus, $B^{-1}A$ is doubly symmetric.

Property 5:The roots of the eigenfilter corresponding to a distinct maximum eigenvalue, lie on the unit circle for a toeplitz $\mathbf{R}_{\underline{x}}(p,q) = \mathbf{R}_{\underline{x}}(k)$

Since the matrix $B^{-1}A$ is doubly symmetric, we can incorporate a proof similar to the one given in [3] to establish this.

Property 6 [2]: The eigenvectors associated with $B^{-1}A$ satisfy either,

$$\begin{aligned} \mathbf{J}\underline{w} &= \underline{w} \quad \text{symmetric} \\ \mathbf{J}\underline{w} &= -\underline{w} \quad \text{skew-symmetric} \end{aligned} \qquad (15)$$

Property 7[2]: If $\mathbf{Q}$ is doubly symmetric with distinct eigenvalues, then $\mathbf{Q}$ has $\lceil p/2 \rceil$ symmetric eigenvectors, and

$\lfloor p/2 \rfloor$ skew symmetric eigenvectors, where $\lceil x \rceil$ ($\lfloor x \rfloor$) indicates the smallest(largest) integer greater(less) than or equal to x.

A doubly symmetric matrix is not symmetric about the main diagonal, hence the eigenvectors are not mutually orthogonal. However, in light of the present theory we can prove the following theorem.

Theorem 1 *Skew-symmetric and symmetric eigenvectors for doubly symmetric matrices are orthogonal to each other.*

Proof:Let,

$$V_1 = \{\underline{w} : \mathbf{J}\underline{w} = \underline{w}\}$$
$$V_2 = \{\underline{w} : \mathbf{J}\underline{w} = -\underline{w}\} \qquad (16)$$

Now,

$$\mathbf{J}\underline{\nu}_1 = \underline{\nu}_1; \quad \underline{\nu}_1 \in V_1 \qquad (17)$$

then with $\underline{\nu}_2 \in V_2$ we have,

$$\underline{\nu}_2^T \mathbf{J} \underline{\nu}_1 = \underline{\nu}_2^T \underline{\nu}_1 \qquad (18)$$

But,

$$\mathbf{J}\underline{\nu}_2 = -\underline{\nu}_2 \Rightarrow \underline{\nu}_2^T \mathbf{J} = -\underline{\nu}_2^T \qquad (19)$$

using the fact the $\mathbf{J}^T = \mathbf{J}$. Substituting (19) into (18) gives,

$$-\underline{\nu}_2^T \underline{\nu}_1 = \underline{\nu}_2^T \underline{\nu}_1 \Rightarrow \underline{\nu}_2^T \underline{\nu}_1 = 0 \qquad (20)$$

which proves the theorem.

Property 8:From the unit norm property of eigenfilters ($\|\underline{w}^*\|^2 = 1$), and *parsevals* relation, we have

$$\int_{2\pi} |W^*(e^{j\omega})|^2 d\omega = 2\pi \qquad (21)$$

Property 9 (Linear phase): Linear phase filters are important for applications where frequency dispersion due to nonlinear phase is harmful (such as speech processing, data transmission, etc.) [4]. The optimal eigenfilter (5) is a linear phase FIR filter having a constant phase and group delay (symmetric case), or a constant group delay (skew-symmetric case) (15), thus

$$\begin{aligned} w^*(m) &= w^*(M-1-m) \quad \text{symmetric} \\ w^*(m) &= -w^*(M-1-m) \quad \text{skew-symmetric} \\ m &= 0, 1, ..., M-1 \end{aligned} \qquad (22)$$

since $\mathbf{J}$ exchanges the elements of the optimal eigenfilter.

4. SENSITIVITY PERFORMANCE OF EIGENFILTERS

In this section, we present some results pertaining to the sensitivity of the eigenfilter to the length of the room impulse response. Essentially, we design the filter of length $M < L$ (L being the duration of the captured room impulse responses in the two directions), based on the windowed room impulse response with duration $P < L$. We then analyze the performance (8), (9) of the filter to increasing room impulse response length (in other words can we design an eigenfilter with a sufficiently short impulse response whose signal cancellation performance would be reasonably invariant to the duration of the room impulse response). Thus the procedure is,

a) Design the eigenfilter $\hat{\underline{w}}^* \in \Re^{M \times 1}$,

$$\hat{\underline{w}}^* = \underline{e}_{\lambda_{max}[\hat{B}^{-1}\hat{A}]} \qquad (23)$$

with,

$$\begin{aligned} \hat{A} &= \sum_{p=0}^{P-1} \sum_{q=0}^{P-1} h_2(p) h_2(q) \mathbf{R}_{\underline{x}}(p,q) \\ \hat{B} &= \sum_{p=0}^{P-1} \sum_{q=0}^{P-1} h_1(p) h_1(q) \mathbf{R}_{\underline{x}}(p,q) \\ & M \leq P < L \end{aligned} \qquad (24)$$

where, the hat above the matrices in (24) denotes an approximation to the true quantities in (6), and the corresponding eigenfilter (23) is the approximation to (5).

b) Evaluate the performance (8) or (9) of the filter to increased room response duration $M \leq P + \Delta P \leq L$, based on the computed eigenfilter (23).

We shall consider the performance when a) $h_i(n) = h_{i,min}(n) \star h_{i,ap}(n)$, and b) $h_i(n) = h_{i,min}(n); i = 1, 2$. The impulse responses $h_1(n)$, and $h_2(n)$ (comprising of 8192 points) were obtained in a room from microphones placed at a radial distance of 3m, with azimuth angle of ± 30 degrees and elevation of 0 degrees relative to a loudspeaker, and are shown in Fig 1 (a),(b). For the input we shall consider the following simple AR(1) (autoregressive process of unit order) linear model generated by filtering noise with a linear time-invariant filter having a rational system function,

$$x(n) = 0.3x(n-1) + 0.7t(n) \qquad (25)$$

where, the $t(n)$ describes a white noise process of zero mean and unit variance. It is well established from the Yule-Walker equations (with $x(n)$ and $t(n)$ being wide-sense stationary) that the correlation function for (25) satisfies the following relation,

$$r_x(k) = \frac{0.49}{0.91}(-0.3)^{|k|} \quad k = .., -1, 0, 1, .. \qquad (26)$$

The power spectrum $S_x(e^{j\omega})$ is,

$$S_x(e^{j\omega}) = \frac{.49}{|1 - .3e^{-j\omega}|^2} \qquad (27)$$

since $\sigma_t^2 = 1$. The reason for this model (25), is that it allows us to establish a framework for speech based signal cancellation, where AR(p) ($p > 1$) could represent the LPC model of the vocal tract over a short time interval, with unvoiced speech (like s in snow) as the noise-free excitation $t(n)$.

A. $h_i(n) = h_{i,min}(n) \star h_{i,ap}(n)$:

In Fig. 2 we show the performance (9) of the eigenfilter design as a function of the length of the impulse response. The length of the FIR filter was $M = 64$. The performance in each subplot as a function of the impulse response increments is shown, where we chose $\Delta P = \{0\} \cup \{2^k : k \in [7, 12], k \in I\}$, where I denotes the integer set. Thus, Fig. 2(a), represents an eigenfilter of length $M = 64$, designed with duration P of the windowed impulse response to be 64 (after removing the pure delay). The second performance *asterisk* is at $P + \Delta P = 64 + 2^7 = 192$, and is obtained by using (27) in (9). In Fig. (3) and Fig. (4), we show the sensitivity of the eigenfilter for filter length $M = 128$, and $M = 256$ for various windowed room impulse responses.

From the figures, it can be immediately seen that, we obtain a better gain performance with increased filter length. By considering a larger duration room impulse response, we lower the gain but improve its evenness (flatness)–which is important, since a minimal duration filter length with large gain and uniform performance (low sensitivity to the length of the room impulse response) is the requirement.

B. $h_i(n) = h_{i,min}(n)$:

In Figs. (5)-(7), we show the performance of the eigenfilter for various windowed duration of the room response and different filter lengths. The performance (in terms of uniformity and gain) is better than the case mentioned above. This can be understood intuitively from (9), which is an alternative expression for (8). Observe that,

$$\begin{aligned} |H_i(e^{j\omega})|^2 &= |H_{i,min}(e^{j\omega})|^2 \\ \text{where} \quad h_i(n) &= h_{i,min}(n) \star h_{i,ap}(n) \quad (28) \\ \text{and} \quad |H_{i,ap}(e^{j\omega})|^2 &= 1; \forall\omega \end{aligned}$$

Thus, the eigenfilter design using (3) for a WSS process is influenced by the minimum phase room impulse responses. Any other choice of impulse responses (nonminimum phase) in (9) leads to poorer sensitivity performance. Hence, $\underline{w}^*$ is as given by (5), with

$$\begin{aligned} A &= \sum_{p=0}^{L-1} \sum_{q=0}^{L-1} h_{2,min}(p) h_{2,min}(q) \mathbf{R}_{\underline{x}}(p,q) \\ B &= \sum_{p=0}^{L-1} \sum_{q=0}^{L-1} h_{1,min}(p) h_{1,min}(q) \mathbf{R}_{\underline{x}}(p,q) \quad (29) \end{aligned}$$

5. CONCLUSIONS

In this paper we considered the eigenfilter design perspective for "signal cancellation". We introduced its theoretical properties based on simple linear algebra, as well as its linear phase property. We demonstrated that the eigenfilter based on the minimum phase room impulse response exhibits better performance than the actual room response for the AR(1) process. This is due to the explicit dependence on the minimum phase room responses of the eigenfilter.

Clearly, we have not addressed the issue pertaining to the human perception of SPL (loudness), which has a frequency dependence on the signal. We shall explore this issue in the future, along with the eigenfilter performance as a function of the source-receiver positions (which affects the reverberant energy). We also plan to implement the filter for speech and audio based models.

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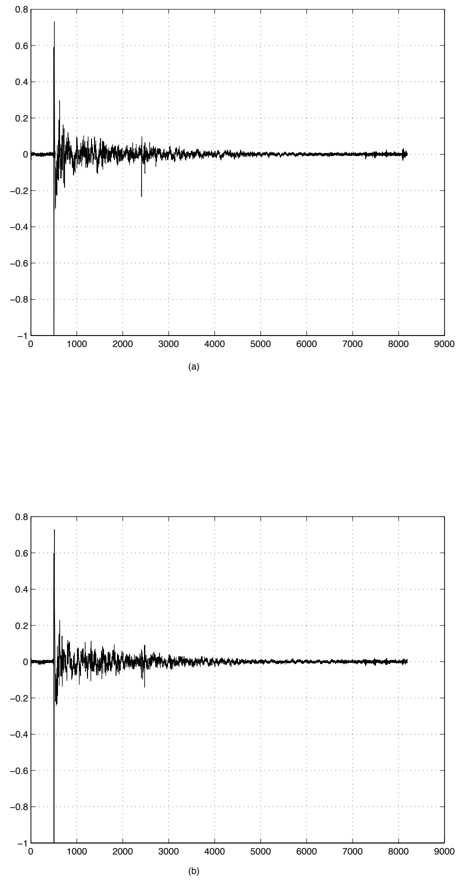

Figure 1: (a), (b) Room impulse responses for given s-r locations

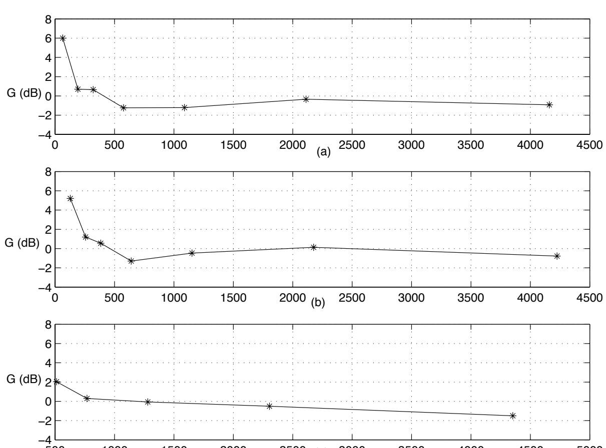

Figure 2: M=64, (a) P=64, (b)P=128, (c)P=512

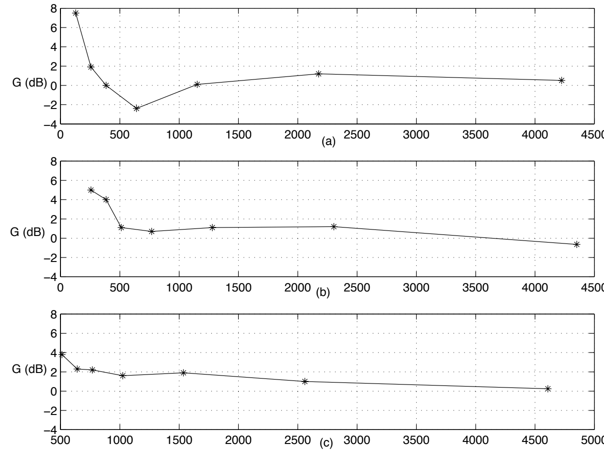

Figure 3: M=128, (a) P=128, (b)P=256, (c)P=512

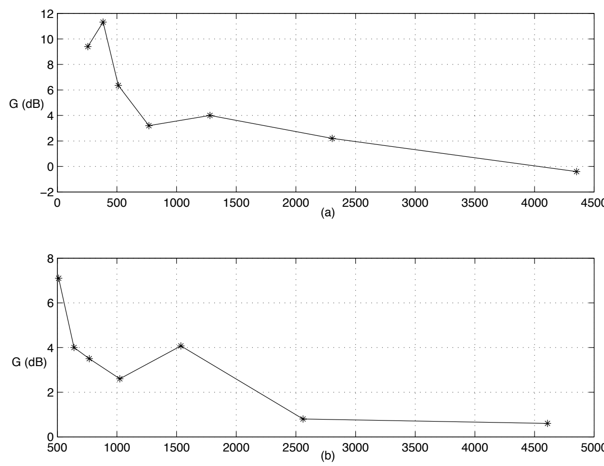

Figure 4: M=256, (a) P=256, (b)P=512

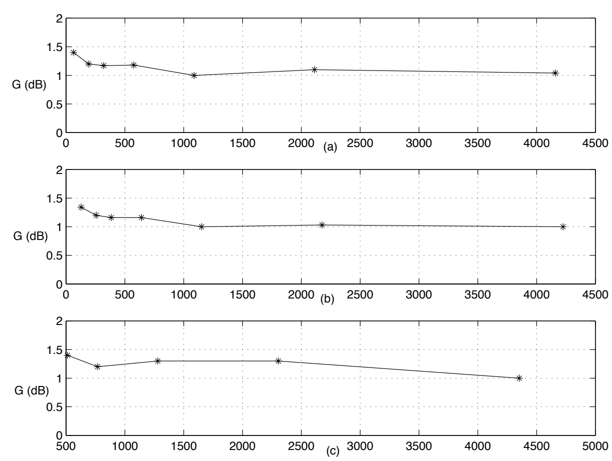

Figure 5: M=64, (a) P=64, (b)P=128, (c)P=512

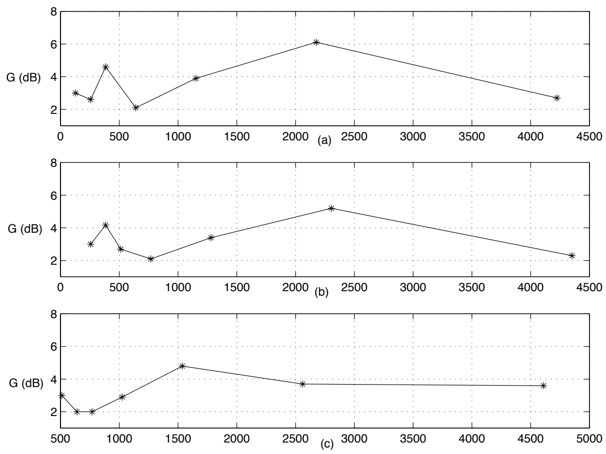

Figure 6: M=128, (a) P=128, (b)P=256, (c)P=512

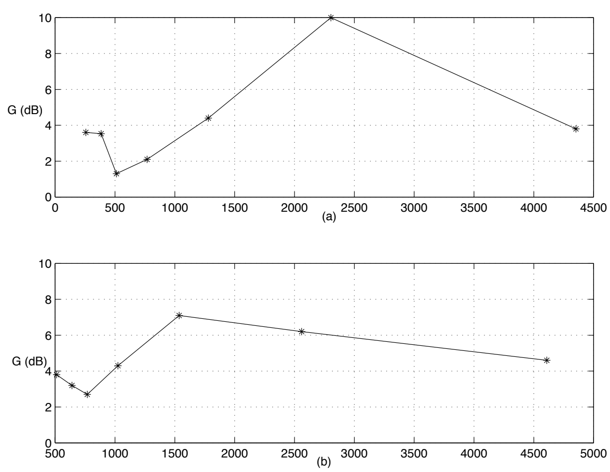

Figure 7: M=256, (a) P=256, (b)P=512